A Network Analysis of Concept Maps of Triangle Concepts

Jin Haiyue
National Institute of Education, Singapore
<haiyue.j@gmail.com>

Wong Khoon Yoong
National Institute of Education, Singapore
<khoonyoong.wong@nie.edu.sg>

Mathematics educators and mathematics standards of curriculum have emphasised the importance of constructing the interconnectedness among mathematic concepts ("conceptual understanding") instead of only the ability to carry out standard procedures in an isolated fashion. Researchers have attempted to assess the knowledge networks in students' minds. A technique that has gained popular use in science education over the past three decades is concept mapping. This paper examines students' conceptual understanding about triangle concepts using concept maps, and an analysis of the maps using degree centralities derived from social network analysis has demonstrated new insights through this novel technique.

Mathematics concepts are logically interconnected. This interconnectedness manifests as coherent knowledge networks, which can be hierarchical or non-hierarchical (web-like). For many years, psychological and educational research on the learning of mathematics has emphasised this interconnectedness as conceptual understanding (Bransford, Brown, & Cocking, 1999; National Council of Teachers of Mathematics, 2000). An important issue concerning this emphasis is how to assess these relationships expressed as cognitive mind maps or knowledge networks so that the information can be used by teachers to plan lessons, and by curriculum developers to take into consideration the psychological, in addition to, the logical knowledge relationships. A technique that has been applied widely in science education over the past three decades is concept mapping.

A concept map can be a "window into the mind" (Shavelson, Ruiz-Primo, & Wiley, 2005, p.1). It is generally defined as a two-dimensional map consisting of nodes representing concepts and labelled lines denoting the relations between pairs of nodes (Novak & Cañas, 2009). These nodes can be mathematical concepts, examples and non-examples of the concepts, diagrams, symbols, and formulas. The labelled lines, also called *linking phrases*, can be verbs or phrases. These labelled lines are usually directional. When two or more concepts are linked, statements are formed, and these statements are called *propositions* (Novak & Gowin, 1984). Thus, a concept map provides an externalised representation in the form of a directed graph of how a person has linked various ideas.

Different methods have been used to interpret the information embedded in concept maps and to score them for assessment purposes. Initially, Novak and Gowin (1984) considered four aspects for scoring: *validity of propositions, hierarchy, cross links*, and *examples*. An updated version includes six criteria to evaluate the map from the concept level to the whole map: "concept relevance and completeness, correct propositional structure, presence of erroneous propositions, presence of dynamic propositions, number and quality of cross links, and presence of cycles" (Novak, 2010, p. 235). Other researchers in mathematics education have described specific aspects of concept maps rather than use systematic coding (see chapters in Afamasaga-Fuata'I, 2009). While these methods have achieved different assessment purposes, more attention could be given to the properties of individual concepts (nodes). For instance, Jin (2007) counted the number of incoming and outgoing links to each node. The incoming links represent the chance to activate this node from other nodes in the concept map; the outgoing links indicate the power of this node to connect to other nodes. However, examining only these attributes of individual nodes does not provide insights about the whole concept map, including dyadic

L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia.* Fremantle: MERGA.

properties of directed links. This paper attempts to fill this gap by applying social network analysis to analyse the properties of the individual nodes and the entire map of concept maps created by students.

Social network analysis (SNA) includes several techniques that use the language of mathematics graph theory to study social relations among people within communities. It uses a variety of attributes such as *centrality, betweenness, closeness,* and *clique* (Degenne & Forsé, 1999; Wasserman & Faust, 1994) to describe such relations. However, we have not found any use of SNA to the study of concept maps. Thus, we attempt to apply SNA to the study of Grade 8 students' concept maps of triangle concepts. Combining this form of analysis with specific discipline (mathematics education in this case) may yield fresh insights about the students' conceptual understanding. In this paper, we will consider only degree centrality analysis, to be defined in a later section.

Methodology

Participants

The participants were 48 Grade 8 students (24 boys and 24 girls) from a junior middle school in Nantong, China. They did not have prior experience of concept mapping.

Training and Concept Map Task

The students first received four 40-minute training sessions on concept mapping. The training, which was developed through several attempts reported in Jin and Wong (2008), was to ensure that the students knew what a concept map was and how to construct meaningful concept maps. Unlike the training programs in traditional concept mapping studies (e.g., Ruiz-Primo, Schultz, Li, & Shavelson, 2001), detailed linking phrases were emphasised in this training. At the end of the training sessions, the students' concept mapping skills (CMS) were tested with a specially designed CMS-Test. Preliminary analysis of the results of the CMS-test indicated that the students had developed the necessary skills to construct meaningful concept maps.

After taking the CMS-test, the students were given a list of eleven concepts related to *triangle.* These concepts were listed in this order (translated from Chinese): *triangle, acute-angled triangle, right-angled triangle, obtuse-angled triangle, scalene triangle, isosceles triangle, equilateral triangle, angle, symmetry axis, median,* and *midline.* These concepts were taken from their Grade 7 mathematics textbook (in Chinese). The students were given a piece of blank paper and were told that they could add extra concepts to their concept maps if they found them related to the given ones. They were allowed to construct either a hierarchical or non-hierarchical map. Thirty minutes were allowed for the students to construct concept maps individually. This was a free-style mapping task (Ruiz-Primo, Shavelson, Li, & Schultz, 2001). This paper reported the network analysis of these student-constructed concept maps.

Data Analysis: Degree Centrality

Degree centrality measures the extent to which a node connects to all other nodes in a network (Knoke & Yang, 2008). In a directed network, there are two separate measures of degree centrality depending on the direction of links: in-degree centrality and out-degree centrality. In-degree centrality of a node counts the number of incoming links directed to the node, and out-degree centrality counts the number of outgoing links from the node

(Durland, 2006). They are defined as IDC (in-degree centrality) and ODC (out-degree centrality):

$$IDC(i) = \sum_{j=1}^{n} x_{ij} (i \neq j) \qquad \text{and} \qquad ODC(i) = \sum_{j=1}^{n} y_{ij} (i \neq j)$$

where x_{ij} is the number of direct links from node j to node i and y_{ij} is the number of direct links from node i to node j. Normally, x_{ij} equals to 0 or 1, and the same for y_{ij}. Thus, $\sum_{j=1}^{n} x_{ij}$ counts the number of incoming links from the other (n – 1) nodes to node i and $\sum_{j=1}^{n} y_{ij}$ counts the number of outgoing links from node i to the other (n – 1) nodes.

The total degree centrality (DC) of node i is then defined as $DC(i) = IDC(i) + ODC(i)$. These degree centralities reflect the connectivity of an individual node to other nodes in a network. For different networks consisting of the same nodes, the in-degree and out-degree centralities allow comparison of a node's connectivity across the networks. The larger the in-degree centrality, the higher is the popularity of a node in a network, and the larger the out-degree centrality, the higher is the influence of a node in the network (Durland, 2006).

Definitions of degree centrality can be extended to the whole network. Freeman (1979, cited in Wasserman & Faust, 1994) proposed a generic measure of *group degree centrality* (GDC) for undirected network with n nodes, which was afterwards revised by Wasserman and Faust (1994, p.180) as:

$$GDC = \frac{\sum_{i=1}^{n} [DC(N^*) - DC(i)]}{(n-1)(n-2)}$$

where DC(i) refers to degree centrality of node i in the undirected network, DC(N*) denotes the largest degree centrality observed in the network, and the denominator is the theoretically maximum possible sum of those differences.

Group degree centrality is used to measure the extent to which the nodes in a network differ from one another in their individual degree centrality. The larger the group degree centrality, the more uneven is the degree centrality of the nodes in a network (Knoke & Yang, 2008).

The above measure of group degree centrality can be extended to directed networks by considering the directions of the links. Group in-degree centrality (GIDC) and group out-degree centrality (GODC) are defined as follows:

$$GIDC = \frac{\sum_{i=1}^{n} [IDC(N^*) - IDC(i)]}{(n-1)^2} \qquad \text{and} \qquad GODC = \frac{\sum_{i=1}^{n} [ODC(N^*) - ODC(i)]}{(n-1)^2}$$

where IDC(i) refers to in-degree centrality of node i in a directed network, ODC(i) refers to out-degree centrality of node i in the network, and IDC(N*) and ODC(N*) respectively denote the largest in-degree centrality and out-degree centrality observed in the network. The denominator refers to the maximum possible sum of differences of in-degree or out-degree centrality. Its value, however, is different from the maximum possible degree centrality for undirected network as defined in *GDC*. For directed graphs, the maximum out-degree centrality occurs when one particular node has an outgoing link to every other nodes but the other nodes do not have any outgoing links, giving the value of (n– 1) – 0 = n – 1. This is repeated (n – 1) times, so the value of the denominator for *GODC* equals to (n – 1)(n – 1). The same value applies to *GIDC.* A higher *GODC* indicates more uneven influence among the nodes in a network, while a higher *GIDC* indicates greater inequality among the nodes' popularity.

A student's concept map is now used to illustrate the above definitions. In this study, the student manually constructed a concept map in Chinese. For ease of presentation, the concept map was translated into English and re-drawn using the software Cmap Tools (available at: http://cmap.ihmc.us), as shown in Figure 1.

The concept *triangle* in Figure 1 was linked to 7 out of 10 concepts in the concept map: its out-degree centrality equals to 7.0. However, there were no links from the 7 concepts to *triangle,* so its in-degree centrality is 0.0.

The concept *equilateral triangle* in Figure 1 had two incoming links and one outgoing link; hence, its in-degree centrality is 2.0 and its out-degree centrality is 1.0.

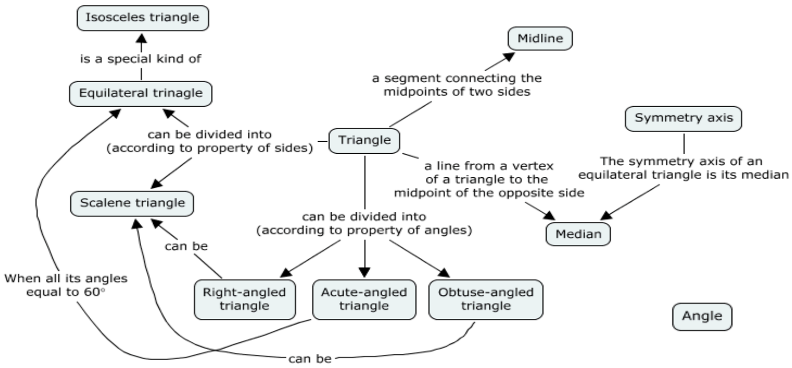

Figure 1. A student-constructed concept map (re-drawn).

The in-degree centrality and out-degree centralities for all the 11 concepts in Figure 1 are given in Table 1. The result shows that *triangle* is the most influential node since it reaches most number of other nodes directly while the other nodes have very low *ODC.* The most popular node identified by *IDC* is *scalene triangle* since it receives the most number of incoming links from the other nodes, although the number (3) is still quite small. Furthermore, the low range of values in *IDC* shows that the difference of the nodes in popularity is relatively small. The concept *angle* is an isolated node in the concept map with zero centralities; it has no link with any of the other nodes.

The total degree centrality in the last column is the total number of the incoming and outgoing links for each node. This score reflects the extent to which a node is connected within a concept map, ignoring the direction of the links. The result shows that *triangle* is well connected in the concept map in Figure 1 and *scalene triangle* and *equilateral triangle* are moderately connected. *Acute-angled triangle, right-angled triangle,* and *obtuse-angled triangle* have the same degree centralities; this suggests that these three types of triangles are of the same connectedness with the nodes in the concept map. The relatively low total degree centralities of the remaining five concepts indicate either that the concepts are mathematically less connected with the other concepts, or that the student was not familiar with the concepts. For example, *isosceles triangle* should have more or less the same number of links as *scalene triangle* and *equilateral triangle* since these three types of triangles are defined according to properties about sides, yet it has relatively low centralities compared to the other two types of triangle. Thus, this student may have an incomplete understanding of *isosceles triangles*.

Table 1
In-degree Centralities and Out-degree Centralities of Nodes in Figure 1

Centrality	Out-degree (ODC)	In-degree (IDC)	Total degree centrality (DC)
Triangle	7.0	0.0	7.0
Acute-angled Triangle	1.0	1.0	2.0
Right-angled Triangle	1.0	1.0	2.0
Obtuse-angled Triangle	1.0	1.0	2.0
Scalene Triangle	0.0	3.0	3.0
Isosceles Triangle	0.0	1.0	1.0
Equilateral Triangle	1.0	2.0	3.0
Symmetry Axis	1.0	0.0	1.0
Angle	0.0	0.0	0.0
Median	0.0	2.0	2.0
Midline	0.0	1.0	1.0
Group degree centrality:	0.650	0.210	0.589

The group degree centralities in the last row cannot be interpreted in isolation. Nevertheless, the *GODC* value of 0.650 reflects the large variation in individual out-degree centralities of the nodes: 7 for *triangle* and 0 or 1 for the rest of the nodes. For this student, *triangle* is the sole central concept from which to link to the other concepts. This could arise because *triangle* was the first item of the given list of concepts or it is the most inclusive concept or the super-concept of the list. See further analysis later on.

Findings: Centralities and Conceptual Understanding

Centralities

The above analysis of a single concept map is now extended to the whole group of 48 students. The mean centralities of the whole group are given in Table 2. For ease of comparison, the list of concepts is sorted by the out-degree centralities, followed by the in-degree centralities, instead of in the order the concepts appear in the given list.

The degree centralities in Table 2 show that *triangle* has the highest out-degree centrality but the lowest in-degree centrality. This suggests that *triangle* is the most influential but also the least easily accessible concept (in terms of the direction of links) among the given set of concepts. The high out-degree centrality (6.98) and low in-degree centrality (0.08) together indicate that *triangle* is less dependent on the other concepts in the concept maps (Hannerman & Riffle, 2005).

At the other end, the concepts *symmetry axis, angle, median,* and *midline* have very low out-degree centralities but average to high in-degree centralities. These four concepts are fairly popular as incoming links so they tend to appear at the end of a conceptual chain. They are not so influential in terms of outgoing links. The concepts *acute-angled triangle, right-angled triangle,* and *obtuse-angled triangle* have average in-degree and out-degree centralities, ranging from 1.10 to 1.92. These values suggest that the concepts are relatively popular with incoming links while at the same time are influential at an intermediate level. However, most of the incoming links to them are from *triangle,* while most of their outgoing links are to *symmetry axis, angle, median,* and *midlines.* Of these three concepts, *right-angled triangle* has the highest out-degree centrality. A possible reason is that, in addition to angle properties, *right-angled triangle* has other properties, which are not shared by the other triangles, hence, increasing its out-degree centrality. For example, the ***median*** to the ***hypotenuse of a right-angled triangle*** equals to half the length of the hypotenuse, and this may have added more outgoing links to *median.*

Table 2
Mean Centralities of Concepts of 48 Student-constructed Concept Maps

Centrality	Out-degree (ODC)	In-degree (IDC)	Total degree centrality (DC)
Triangle	6.98	0.08	7.06
Equilateral Triangle	2.73	1.48	4.21
Isosceles Triangle	2.04	2.19	4.23
Right-angled Triangle	1.92	1.35	3.27
Acute-angled Triangle	1.27	1.81	3.08
Obtuse-angled Triangle	1.10	1.27	2.37
Scalene Triangle	1.00	1.65	2.65
Angle	0.27	2.81	3.08
Median	0.21	1.48	1.69
Midline	0.17	1.44	1.61
Symmetry Axis	0.10	2.23	2.33

Among the other three types of triangles, *scalene triangle, isosceles triangle,* and *equilateral triangle*, *scalene triangle* have the lowest out-degree centrality and *isosceles triangle* has the highest in-degree centrality. This suggests that these students had more knowledge about *isosceles triangle* and *equilateral triangle* than about *scalene triangle,* since the first two types of triangles have more special properties not shared by *scalene triangle.* For example, *isosceles triangle* and *equilateral triangle* have *symmetry axis* and equal *angles*, while these properties are not shared by *scalene triangle*. Thus, *scalene triangle* has fewer outgoing links.

The above impressions can also be gained from a consideration of the group total-degree centralities, which range from 1.61 (*midline*) to 7.06 (*triangle*), suggesting that the concepts are of different connectedness levels. Logically, *triangle* is the most inclusive of the given set of concepts. The six types of triangles are located at a medium level since they are less inclusive than *triangle* but more general than *symmetry axis, median,* and *midline*. There are differences among these types of triangles: the students had fewer links related to *scalene triangle* and *obtuse-angled triangle*, compared to the other four types of triangles, which are more common. The remaining three concepts, *symmetry axis, median,* and *midline,* are special properties of triangles, thus, residing at lower levels of "hierarchy". *Angle* is also a generic concept, but its out-degree centrality is very low, suggesting that these students did not see how *angle* can lead to the other concepts, as illustrated by its isolation in Figure 1. The above degree centrality analysis is consistent with the attributes of the concepts based on logical considerations. This supports the use of this type of analysis for concept maps to probe student conceptual understanding at an individual level (Table 1) as well as group level (Table 2).

Centralities and Mathematics Scores

As shown above, each concept map can be characterised by two group degree centralities, *GIDC* and *GODC*. The following correlation analysis examines the relations between the degree centralities and the students' school mathematics achievement.

Results from six school mathematics tests were collected. These six tests were two final mathematics tests of the two semesters in Grade 7, the mid-term mathematics test and the final mathematics test of the first semester in Grade 8, and the mid-term mathematics test and a monthly mathematics test within the data collection period in the second semester of Grade 8. These tests measured achievement in several topics such as *equations* and *quadrilateral*, but their correlations ranged from 0.926 to 0.950, all significant at the 0.001 level (2-tailed). Thus, the average score of the six tests was taken as an indicator or proxy of the student's School Mathematics Achievement (SMA).

To address the fact that the six tests covered different topics, a specially designed conceptual understanding test (CU-test) on *triangle* was administered one day before the concept map task. This CU-test (triangle) was designed according to the first three levels of van Hiele's theory of geometric understanding, i.e., *visualisation*, *analysis*, and *abstraction*. The items cover definitions and properties of triangles as well as their relationships. Unlike common school mathematics tests that are mainly about solving problems, the CU-test assesses conceptual understanding. As shown in Table 3, this CU-test was also strongly related to SMA, indicating that both measure some underlying mathematics achievement. The results of the correlation analyses are shown in Table 3.

Table 3.
Correlation Coefficients between Degree Centralities and Mathematics Tests

Centralities	Group out-degree (GODC)	Group in-degree (GIDC)	Group total-degree (GDC)	School math achievement (SMA)
Group in-degree centrality (GIDC)	0.156			
Group total-degree centrality (GDC)	0.866**	0.629**		
School math achievement (SMA)	0.357*	0.470**	0.153**	
CU-test (triangle)	0.550**	0.340*	0.338**	0.815**

* $p < 0.05$ and ** $p < 0.001$.

No statistically significant relation was found between group out-degree centrality and group in-degree centrality. Thus, a concept that is "influential" in terms of its outgoing links to other concepts may or may not be "popular" in terms of its incoming links. The correlations between the in-degree and out-degree centralities and SMA and CU-test ranged from 0.340 to 0.550, which are significant at the 0.05 or 0.001 levels. Besides, the group total-degree centrality had higher correlation coefficients with CU-test (0.338, p=0.019) than with SMA (0.153, p=298). This shows that the degree centralities might relate more to students' conceptual understanding than problem solving since CU-test emphasises conceptual understanding whereas the school mathematics tests were about problem solving.

Conclusion

This investigation supports the idea that degree centralities from SNA can be adopted for analysing concept maps. The degree centralities provide information about the connectedness of the individual concepts within concept maps, which is not easily detected with traditional methods such as scoring rubrics and anecdotal descriptions. The analysis can be readily completed through counting and simple calculations, and this ease of use will be an advantage to researchers and teachers who are contemplating using a concept map as an additional assessment tool. The concurrent validity of degree centralities for assessing student conceptual understanding has also been demonstrated through correlation analysis between these attributes and mathematics achievement (in particular on CU-test) reported above.

SNA has other techniques not discussed in this paper. Further research can also investigate concept maps using other SNA measures such as *closeness* centrality and *betweenness* centrality. These measures allow examination of the links in concept maps from multiple views to ensure a fuller understanding of concept maps as well as their relations with students' conceptual understanding and mathematics achievement.

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